

Tutorial 9 - Symmetric groups

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Problem 1. Show that the number of all tabloids for given $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$ is $\binom{n}{\lambda_1, \lambda_2, \dots, \lambda_k}$.

Problem 2. Describe the Specht module S^λ and the action of S_n on it explicitly for $\lambda = (n)$, $\lambda = (1, 1, \dots, 1)$ and $\lambda = (n-1, 1)$ and compute their dimension.

Problem 3. Compute the number of tableau for a given λ : $(3, 2, 1, 1)$, (n) , $(n-1, 1)$, $(n-2, 2)$.

Problem 4. Compute the character table for S_5 .

Problem 5. Determine the chromatic symmetric function for a graph $G: P_3, P_2 + P_2 + P_1, K_3 \wedge P_2$ and K_n .

Problem 6. Suppose we restrict S^λ to S_{n-1} . Then it holds that this restriction is a direct sum of S^{λ^-} , where λ^- denotes Young diagram obtained by deleting one box from λ .

a) Decompose the restriction $\text{Res}_{S_3}^{S_4} S^{(2,2)}$,

b) Decompose $\text{Res}_{S_5}^{S_6} S^{(3,2,1)}$,

c) Use this rule to prove that the number of SYT for given λ is equal to $\dim S^\lambda$.

Problem 7. Express the following polynomials in basis of elementary symmetric and power sum symmetric polynomials:

a) $x^2yz + xy^2z + xyz^2$,

b) $x^3y + xy^3 + 2x^2y^2 + x + y$,

c) $x^3(y+z) + y^3(x+z) + z^3(x+y)$,