Tutorial 8 - Characters

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Problem 1. Let φ be an irreducible character of G and let χ be a one-dimensional character of G. Prove that $\omega = \varphi \otimes \chi$ is an irreducible character.

Problem 2. Describe the conjugacy classes of the following groups: S_4, A_4, D_{10}, $\mathbb{Z}_8$.

Problem 3. Show that two permutations $\sigma, \pi \in S_n$ are conjugated if and only if $\lambda(\sigma) = \lambda(\pi)$.

Problem 4. Let χ be a non-trivial irreducible character of a finite group G. Show that

$$\sum_{g \in G} \chi(g) = 0.$$

Problem 5. Let $\varphi\colon G \to \mathbf{GL}(V)$ be a representation of a finite group G. We define a *fixed subspace* as

$$V^G = \{v \in V | \varphi_g v = v \, \forall g \in G\}.$$

a) Observe that V^G is an invariant subspace of V.

b) Show that $\frac{1}{|G|} \sum_{h \in G} \varphi_h v \in V^G$ for all $v \in V$.

c) Show that if $v \in V^G$, then $\frac{1}{|G|} \sum_{h \in G} \varphi_h v = v$.

Problem 6. Compute the character table for $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Problem 7. Compute the character table for S_4.