

Tutorial 7 - Representations of finite groups

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Problem 1. Let $\varphi: D_8 \rightarrow \mathbf{GL}_2(\mathbb{C})$ be the representation given by

$$\varphi(r^k) = \begin{pmatrix} i^k & 0 \\ 0 & (-i)^k \end{pmatrix}; \quad \varphi(sr^k) = \begin{pmatrix} 0 & (-i)^k \\ i^k & 0 \end{pmatrix}$$

where r is a rotation counterclockwise by $\frac{\pi}{2}$ and s is reflection over the x -axis. Prove that φ is irreducible and compute its character.

Problem 2. Let $\varphi, \psi: G \rightarrow \mathbb{C}^*$ be one-dimensional representations. Show that φ is equivalent to ψ if and only if $\varphi = \psi$.

Problem 3. Let G be a finite abelian group. Show that all irreducible representations are exactly characters.

Problem 4. Let ρ and τ be representations of G over the same field. Show that $\chi(\rho \oplus \tau) = \chi(\rho) + \chi(\tau)$ and $\chi(\rho \otimes \tau) = \chi(\rho) \cdot \chi(\tau)$.

Problem 5. Let $\varphi: G \rightarrow \mathbf{GL}_n(\mathbb{C})$ and $\chi: G \rightarrow \mathbb{C}^*$ be representations of G . Define a map $\varphi^\chi: G \rightarrow \mathbf{GL}_n(\mathbb{C})$ by $\varphi_g^\chi = \chi(g)\varphi_g$. Show that φ^χ is a representation. Give an example showing that φ and φ^χ do not have to be equivalent.

Problem 6. Compute the character table for symmetric group S_3 .

Problem 7. Find a representation $\rho: G \rightarrow \mathbf{GL}(V)$ with an invariant subspace $W < V$ such that W^\perp is not invariant.

Note: You are invited to spring school of algebra. More here: <https://karlin.mff.cuni.cz/~skolavpriode/en>.