Tutorial 1 - rewind of algebra, characters

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Problem 1. Show that all roots of $x^n - 1$ are of the form $e^{\frac{2k\pi i}{n}}$ for some $k \in \mathbb{Z}$.

Problem 2. Let G, H be groups and $\varphi\colon G \to H$. Show that for each $g \in G$ it holds that $\operatorname{ord}(\varphi(g)) \mid \operatorname{ord}(g)$.

Problem 3. Let $z \in \mathbb{T}$. Show that $z^{-1} = \overline{z}$.

Problem 4. Show that if χ is a character of G, then also $\frac{1}{\chi}$ is a character of G.

Problem 5. Show that every homomorphism $\chi\colon G \to \mathbb{C}^*$ is a character.

Problem 6. Show that $\mathbb{T}$ and $\mathbb{R}/\mathbb{Z}$ are isomorphic.

Problem 7. Show that $\mathbb{C}G = \{f\colon G \to \mathbb{C}\}$ is a vector space over $\mathbb{C}$ with inner product $\langle f, g\rangle = \frac{1}{|G|}\sum_{x\in G} f(x)\overline{g(x)}$.

Problem 8. Show that $\mathbb{C}\mathbb{Z}_n$ has an orthonormal basis $(\delta_i)_{i=0}^{n-1}$, where $\delta_j(i) = 1$ if $i = j$ and 0 otherwise.

Problem 9. Let $f \in \mathbb{C}\mathbb{Z}_n$. Show that $T\colon \mathbb{C}\mathbb{Z}_n \to \mathbb{C}^n$, where $T(f) = (f(0), f(1), \ldots, f(n-1))$, is a vector space isomorphism.

Problem 10. Let $f\colon \mathbb{Z}_2^n \to \mathbb{Z}_2$ be a map. Then the influence of the k-th variable on f is $\operatorname{Inf}_k(f) = \mathbb{P}[x \in \mathbb{Z}_2^n\colon f(x) \neq f(x + e_k)]$. Compute $\operatorname{Inf}_k(x_1)$ and $\operatorname{Inf}_k(\sum_{i=1}^n x_i)$.

Problem 11. Show that $\widehat{\bigoplus_{i=1}^n G_i} \cong \bigoplus_{i=1}^n \widehat{G_i}$.