Exercise 1 – Eukleidés and Bézout

September 30, 2026

Problem 1. Find the following values:

a) $\mathrm{GCD}(104, 168)$, $\mathrm{LCM}(104, 168)$,

b) $\mathrm{GCD}(728, 1001)$, $\mathrm{LCM}(728, 1001)$,

c) $\triangleleft$ $\mathrm{GCD}(F_n, F_{n+1})$, $\mathrm{LCM}(F_n, F_{n+1})$,

d) $\mathrm{GCD}(2k + 1, 3k - 1)$, $\mathrm{LCM}(2k + 1, 3k - 1)$.

Problem 2. $\triangleleft$ A supermarket sells milk chocolate, white chocolate, and dark chocolate, all at the same price.

On one particular day, the revenue from the milk chocolate sold was 270, from the white chocolate 189, and from the dark chocolate 216. What is the smallest possible number of chocolate bars that could have been sold that day?

Problem 3. Determine the Bézout coefficients for the following pairs:

a) 1023, 96;

b) 104, 168;

c) $2k + 1$, $3k - 1$.

Problem 4. Find the integer solutions to the following equations:

a) $1023x + 96y = 18$,

b) $18x + 24y = 5$,

c) $18x + 24y = 12$.

Problem 5. Compute 10^{-1} in $\mathbb{Z}_{37}$.

Problem 6. Compute 27^{-1} in $\mathbb{Z}_{41}$.

Problem 7. Compute 8^{-1} and 12^{-1} in $\mathbb{Z}_{27}$. Think about why this is the case.

Problem 8. Solve the following congruences:

a) $5 - 3x \equiv 4 \mod 7$,

b) $11 + 4x \equiv 7 \mod 12$,

c) $4x + 7 \equiv 3 - 6x \mod 8$.

Problem 9. Show that $n^2 \equiv 1 \mod 8$ for all odd $n \in \mathbb{N}$.

Problem 10. Solve the following equation: $x^2 + 5x \equiv 0 \mod 19$.

Problem 11. Solve the following equation: $x^2 + 10x + 6 \equiv 0 \mod 17$.

Problem 12. Using modular arithmetic, derive the divisibility criteria for 9 and 11.

Homework. Find the largest number n such that, by subtracting 2020 and adding n, we can reach any integer from 0 to 8787 without ever leaving the interval $[0, 8787]$, regardless of the number we start with.

Hint: use Bézout's theorem to create 1.